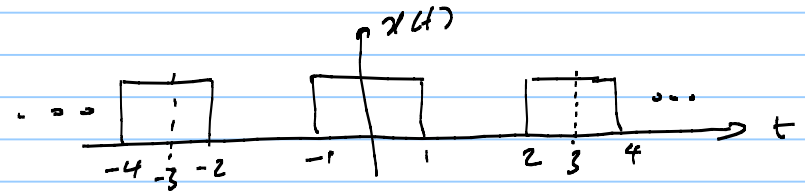
Note Title 3/12/2019

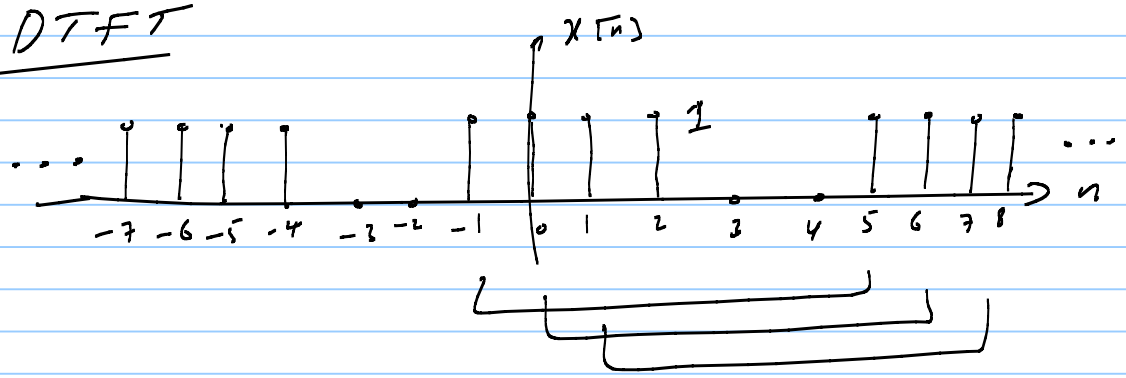
Fourier Transform of periodic signals.

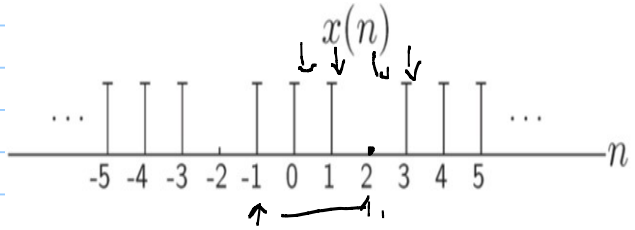


$x(t-c) \leftrightarrow X(\omega) e^{-j\omega c}$

$x(t) e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0)$

DTFT





- $a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\Omega_0 n}$

$\Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{4} = \frac{\pi}{2}$

$a_k = \frac{1}{4} \sum_{n=0}^{3} x[n] e^{-jk\frac{\pi}{2}n}$

$= \frac{1}{4} \left[x[0] e^{0} + x[1] e^{-j\frac{k\pi}{2}} + x[2] e^{-jk\pi} + x[3] e^{-jk\frac{3\pi}{2}} \right]$

$a_k = \frac{1}{4} \left[1 + e^{-j\frac{k\pi}{2}} + 0 + e^{-j\frac{3k\pi}{2}} \right]$

$= \frac{1}{4} \left[1 + \cos\left(\frac{k\pi}{2}\right) - j\sin\left(\frac{k\pi}{2}\right) + \cos\left(\frac{3k\pi}{2}\right) - j\sin\left(\frac{3k\pi}{2}\right) \right]$

$\cos\left(3k\frac{\pi}{2}\right) = \cos\left(3k\frac{\pi}{2} - 2\pi k\right) = \cos\left(-k\frac{\pi}{2}\right) = \cos\left(k\frac{\pi}{2}\right)$

$$\sin\left(3\frac{k\pi}{2}\right) = \sin\left(3\frac{k\pi}{2} - 2\pi k\right) = \sin\left(-\frac{k\pi}{2}\right) = -\sin\left(\frac{k\pi}{2}\right)$$

$$a_k = \frac{1}{4}\left[1 + \cos\left(\frac{k\pi}{2}\right) - j\sin\left(\frac{k\pi}{2}\right) + \cos\left(\frac{k\pi}{2}\right) + j\sin\left(\frac{k\pi}{2}\right)\right]$$

$$a_k = \frac{1}{4}\left[1 + 2\cos\left(\frac{k\pi}{2}\right)\right]$$

$k=0$ $\quad k=1$

$$a_0 = \frac{3}{4} \; ; \; a_1 = \frac{1}{4} \; ; \; a_2 = -\frac{1}{4} \; ; \; a_3 = \frac{1}{4}$$

$$x[n] = \sum_{k=0}^{3} a_k e^{jk\Omega_0 n}$$

- $x[n] = \sum_{k=0}^{N-1} a_k e^{jk\Omega_0 n}$

$$= a_0 e^{0} + a_1 e^{j\frac{\pi}{2}n} + a_2 e^{j\pi n} + a_3 e^{j\frac{3\pi}{2}n}$$

$$= \frac{3}{4} + \frac{1}{4} e^{j\frac{\pi}{2}n} - \frac{1}{4} e^{j\pi n} + \frac{1}{4} e^{j\frac{3\pi}{2}n} \leftarrow$$

$$= \frac{3}{4} + \frac{1}{4}\left\{\cos\left(\frac{\pi}{2}n\right) + j\sin\left(\frac{\pi}{2}n\right)\right\}$$
$$- \frac{1}{4}\left\{\cos(\pi n) + j\sin(\pi n)\right\}$$
$$+ \frac{1}{4}\left[\cos\left(3\frac{\pi}{2}n\right) + j\sin\left(3\frac{\pi}{2}n\right)\right]$$

$$\sin\left(\frac{3\pi}{2}n\right) = \sin\left(\frac{3\pi}{2}n - 2\pi n\right) = \sin\left(-\frac{\pi}{2}n\right) = -\sin\left(\frac{\pi}{2}n\right)$$

$$\sin(\pi n) = 0$$

$$\cos\left(3\frac{\pi}{2}n\right) = \cos\left(\frac{\pi}{2}n\right) \; ; \; \cos(\pi n) = (-1)^n$$

$$\boxed{x[n] = \frac{3}{4} + \frac{1}{2}\cos\left(\frac{\pi}{2}n\right) - \frac{1}{4}(-1)^n}$$

$$x[0] = 1 \; ; \; x[1] = 1 \; ; \; x[2] = 0 \; ; \; x[3] = 1$$

$$\cos\left(\frac{3\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) \; ; \; \sin\left(\frac{3\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right)$$

$$X(\Omega) = \sum_{n=-\infty}^{n=\infty} x[n] e^{-j\Omega n}$$

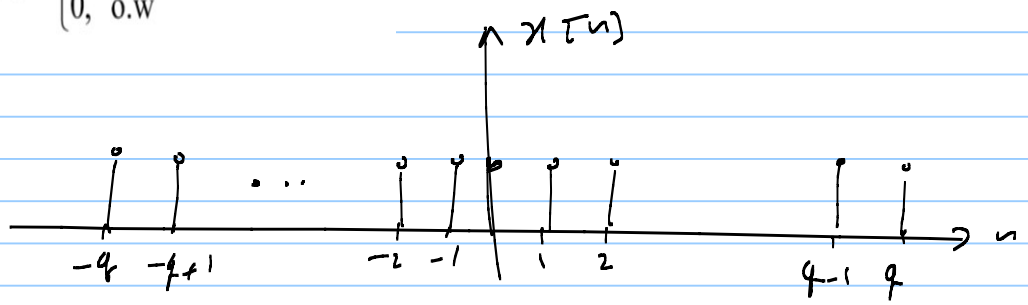
$$X(\Omega + 2\pi) = \sum_{n=-\infty}^{\infty} x[n] e^{-j(\Omega + 2\pi)n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} \underbrace{e^{-j2\pi n}}_{1}$$

$$e^{-j2\pi n} = \cos(2\pi n) - j\sin(2\pi n) = 1$$

$$\therefore X(\Omega + 2\pi) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = X(\Omega)$$

$$x[n] = \begin{cases} 1, & n = -q, -q+1, \ldots, q-1, q \\ 0, & \text{o.w} \end{cases}$$



$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{n=-q}^{q} e^{-j\Omega n} \xrightarrow{k=n+q} \sum_{k=0}^{2q} e^{-j\Omega(k-q)}$$

$$= \sum_{k=0}^{2q} e^{-jk\Omega} e^{jq\Omega} = e^{jq\Omega} \sum_{k=0}^{2q} e^{-jk\Omega}$$

$$\sum_{i=0}^{n} a^i = \frac{1-a^{n+1}}{1-a}$$